7 TRIANGLES

Congruence of Triangles

Congruent Figures

↓

Only when they have exactly

↓

Same size and same shape

- ← Two equal angles
- → Two line segments of equal length
- ↓ Circles of same radii
- ↓ Two triangles if they have equal corresponding sides and equal corresponding angles
- ↓ Squares of same sides

Two triangles if they have equal corresponding sides and equal corresponding angles

↓

Example: In ΔABC and ΔDEF,
If AB = DE, BC = EF, CA = FD
and ∠A = ∠D, ∠B = ∠E, ∠C = ∠F,
then ΔABC ≅ ΔDEF

↓

Criteria for congruence of triangles

SAS (Side-Angle-side) Congruence rule

↓

Two sides and the included angle

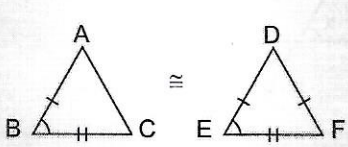
↓

In ΔABC and ΔDEF,
- AB = DE, ∠B = ∠E

and BC = EF

↓

ΔABC ≅ ΔDEF



ASA (Angle-Side-Angle) Congruence rule

↓

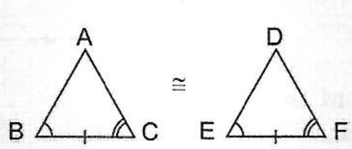
Two angles and included side

↓

In ΔABC and ΔDEF,
- ∠B = ∠E, BC = EF
- ∠C = ∠F

↓

ΔABC ≅ ΔDEF



AAS (Angle-Angle-Side) Congruence rule

↓

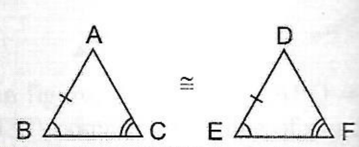
Two angles and one pair of corresponding sides

↓

In ΔABC and ΔDEF,
∠B = ∠E, ∠C = ∠F
AB = DE or AC = DF

↓

ΔABC ≅ ΔDEF



Note: (*i*) In congruent triangles, corresponding parts are equal.

(*ii*) It is necessary to write the correspondence vertices correctly for writing of congruence of triangles.

(*iii*) If ΔABC ≅ ΔDEF, then it also be written as
ΔBCA ≅ ΔEFD or ΔCAB ≅ ΔFDE; but not as ΔABC ≅ ΔEFD or ΔBCA ≅ ΔFDE.

(*iv*) CPCT means corresponding parts of congruent triangles.

(*v*) SAS congruence rule holds, but not ASS and SSA rule.

(*vi*) The sum of the three angles of a triangle is 180°. So if two pairs of angles are equal, the third pair is also equal.

SOLVED QUESTIONS BASED ON EXERCISE 7.1

Very Short Answer Type Questions [1 Mark]

1. In ΔABC and ΔDEF, AB = FD and ∠A = ∠D. Write the third condition for which two triangles are congruent by SAS congruence rule. **[CBSE 2011]**

Sol. By SAS congruence rule, the arms of equal angles must also be equal.

Hence, AB = FD

∠A = ∠D

So AC = DE

⇒ ΔABC ≅ ΔDFE (SAS congruence rule)

2. It is given that ΔABC ≅ ΔFDE and AB = 6 cm, ∠B = 40° and ∠A = 80°. What is length of side DF of ΔFDE and its ∠E?

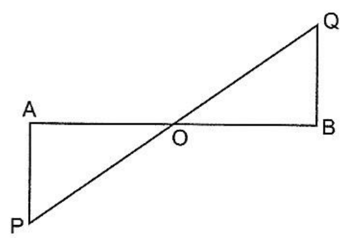
Sol. **Given:** ΔABC ≅ ΔFDE

Now, corresponding parts of congruent triangles are equal.

So, DF = AB = 6 cm

and ∠E = ∠C = 180° – (80° + 40°) = 60°

3. In the given figure, O is the mid-point of AB and ∠BQO = ∠APO. Show that ∠OAP = ∠OBQ. **[CBSE 2014]**



Sol. **Given:** (*i*) O is mid-point of AB.

(*ii*) ∠BQO = ∠APO

To prove: ∠OAP = ∠OBQ

Proof: In ΔOAP and ΔOBQ,

OA = OB (O is mid-point of AB)

∠APO = ∠BQO (Given)

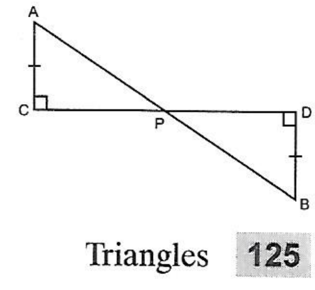
∠AOP = ∠BOQ (Vertically opposite angles)

⇒ ΔOAP ≅ ΔOBQ (ASA congruence rule)

⇒ ∠OAP = ∠OBQ (CPCT)

Hence proved

4. In the given figure, CA and DB are perpendiculars to CD and CA = DB. Show that PA = PB.



Sol. **Given:** (*i*) CA ⊥ CD

(*ii*) DB ⊥ CD

(*iii*) CA = DB

To prove: PA = PB

Proof: In ΔCPA and ΔDPB,

$\angle ACP = \angle BDP$ (Each 90°)

$\angle CPA = \angle DPB$ (Vertically opposite angles)

CA = DB (Given)

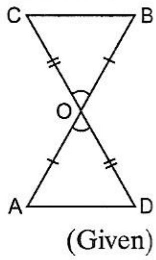
$\Rightarrow$ $\Delta CPA \cong \Delta DPB$ (AAS congruence rule)

$\Rightarrow$ PA = PB (CPCT)

Hence proved

Short Answer Type Questions I [2 Marks]

5. In the given figure, if OA = OB, OD = OC. Prove that $\Delta AOD \cong \Delta BOC$.



Sol. Given: (*i*) OA = OB

(*ii*) OD = OC

To prove: $\Delta AOD \cong \Delta BOC$

Proof: In ΔAOD and ΔBOC,

OA = OB (Given)

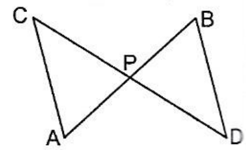
$\angle AOD = \angle BOC$ (Vertically opposite angles)

OD = OC (Given)

$\Rightarrow$ $\Delta AOD \cong \Delta BOC$ (SAS congruence rule)

Hence proved

6. In the given figure, AC = BD and AC||DB. Prove that $\Delta APC \cong \Delta BPD$.



Sol. Proof: Given AC||DB

AB is transversal

$\Rightarrow$ $\angle PAC = \angle PBD$ (Alternate interior angles)

When CD is transversal, then

$\angle PCA = \angle PDB$ (Alternate interior angles)

Now, in ΔAPC and ΔBPD,

$\angle A = \angle B$ (As proved above)

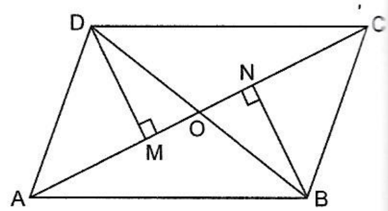
AC = BD (Given)

$\angle C = \angle D$ (As proved above)

$\Rightarrow$ $\Delta APC \cong \Delta BPD$ (ASA congruence rule)

Hence proved

7. In quadrilateral ABCD, BN and DM are drawn perpendicular to AC. Such that BN = DM. Prove that O is mid-point of BD.



Sol. In ΔDMO and ΔBNO,

$\angle DMO = \angle BNO = 90°$ (Given)

$\angle DOM = \angle BON$ (Vertically opposite angles)

DM = BN (Given)

$\therefore$ $\Delta DMO \cong \Delta BNO$ (AAS congruence rule)

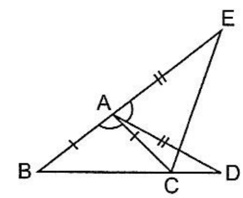
$\Rightarrow$ DO = BO (CPCT)

$\Rightarrow$ O is mid-point of BD

Hence proved

Short Answer Type Questions II [3 Marks]

8. In the given figure, AB = AC, AD = AE and ∠BAC = ∠DAE. Prove that ΔBAD ≅ ΔCAE. **[CBSE 2010]**



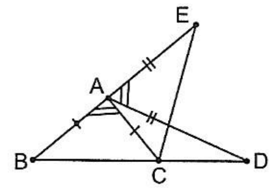
Sol. Given: AB = AC

AD = AE

and ∠BAC = ∠DAE

To Prove: ∠BAD ≅ ΔCAE

Proof: ∠BAC = ∠DAE (Given)



On adding ∠DAC both sides, we get

∠BAC + ∠DAC = ∠DAE + ∠DAC

⇒ ∠BAD = ∠EAC

In ΔBAD and ΔEAC

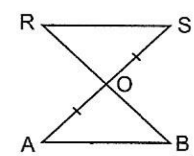
BA = CA (Given)

∠BAD = ∠EAC (Proved above)

AD = AE (Given)

∴ ΔBAD ≅ ΔEAC (SAS congruence rule)

9. In the given figure, the line segment AB is parallel to another line segment RS and O is the mid-point of AS. Show that



(*i*) ΔAOB ≅ ΔSOR

(*ii*) O is mid-point of BR. **[CBSE 2013]**

Sol. Given: (*i*) AB||RS

(*ii*) O is mid-point of AS.

To prove: (*i*) ΔAOB ≅ ΔSOR

(*ii*) O is mid-point of BR.

Proof:

(*i*) **Given:** AB||RS and AS is transversal.

⇒ ∠OAB = ∠OSR (Alternate interior angles)

Now, in ΔAOB and ΔSOR,

∠OAB = ∠OSR (Proved above)

OA = OS (O is mid-point of AS)

and ∠AOB = ∠SOR (Vertically opposite angles)

⇒ ΔAOB ≅ ΔSOR (ASA congruence rule)

Hence proved.

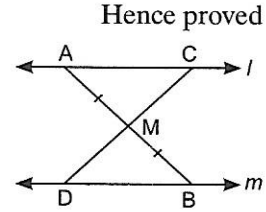
(*ii*) As ΔAOB ≅ ΔSOR,

So OB = OR (CPCT)

⇒ O is mid-point of BR.

Hence proved

10. In the given figure, $l||m$ and M is the mid-point of line segment AB. Prove that M is also the mid-point of any line segment CD having its end points C and D on l and m respectively.



Sol. Given: (*i*) $l||m$

(*ii*) M is mid-point of AB.

To prove: M is mid-point of CD.

Proof: Given $l||m$ and AB is transversal

⇒ ∠CAM = ∠DBM (Alternate interior angles)

Now, in ΔAMC and ΔBMD,

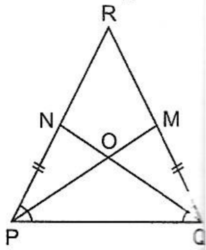
∠CAM = ∠DBM (As proved above)

	AM = BM	(M is mid-point of AB)
	$\angle AMC = \angle BMD$	(Vertically opposite angles)
$\Rightarrow$	$\Delta AMC \cong \Delta BMD$	(ASA congruence rule)
$\Rightarrow$	CM = DM	(CPCT)

$\Rightarrow$ M is mid-point of CD.

Hence proved.

11. In the given figure, $\angle QPR = \angle PQR$ and M and N are respectively points on the sides QR and PR of ΔPQR, such that QM = PN. Prove that OP = OQ, where O is the point of intersection of PM and QN.



Sol. Given: In ΔPQR, $\angle QPR = \angle PQR$ M and N are two points on QR and PR such that QM = PN. PM and QN intersect at O.

To prove: OP = OQ

Proof: In ΔPQM and ΔPQN, we have,

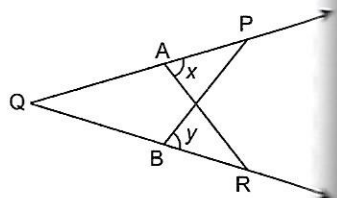
	QM = PN	[Given]
$\because$	$\angle QPM = \angle PQN$	[Given: $\angle QPR = \angle PQR$]
	PQ = PQ	[Common]
$\therefore$	$\Delta PQM \cong \Delta PQN$	[SAS congruence rule]
$\therefore$	$\angle QPM = \angle PQN$	
But	$\angle QPN = \angle PQM$	
$\Rightarrow$	$\angle QPN - \angle QPM = \angle PQM - \angle PQM$	
$\Rightarrow$	$\angle OPN = \angle OQM$	

Again in ΔPON and ΔQOM, we have

	PN = QM	[Given]
$\because$	$\angle OPN = \angle OQM$	[As proved]
	$\angle PON = \angle QOM$	[Vertically opposite angles]
$\therefore$	$\Delta PON \cong \Delta QOM$	[AAS congruence rule]
$\therefore$	OP = OQ	Hence proved

Long Answer Type Questions [4 Marks]

12. In the given figure, PQ = QR and $\angle x = \angle y$. Prove that AR = PB. **[CBSE 2011]**



Sol. Proof: In the figure,

	$\angle QAR + \angle PAR = 180°$	(Linear pair axiom)
$\Rightarrow$	$\angle QAR + \angle x = 180°$	
$\Rightarrow$	$\angle QAR = 180° - \angle x°$	...(i)
Similarly,	$\angle QBP + \angle RBP = 180°$	(Linear pair axiom)
$\Rightarrow$	$\angle QBP + \angle y = 180°$	
$\Rightarrow$	$\angle QBP = 180° - \angle y$	...(ii)
But given,	$\angle x = \angle y$	
$\therefore$	$\angle QAR = \angle QBP$	[From (i) and (ii)]
Now, in ΔQAR and ΔQBP,	QR = PQ	(Given)
	$\angle QAR = \angle QBP$	(As proved above)
	$\angle Q = \angle Q$	(Common)
$\Rightarrow$	$\Delta QAR \cong \Delta QBP$	(AAS congruence rule)
$\Rightarrow$	AR = PB	(CPCT)

Hence proved.

13. Prove that "Two triangles are congruent, if two angles and the included side of one triangle are equal to two angles and the included side of other triangle". **[CBSE 2014, 11]**

Sol. Given: Two triangles ABC and PQR in which

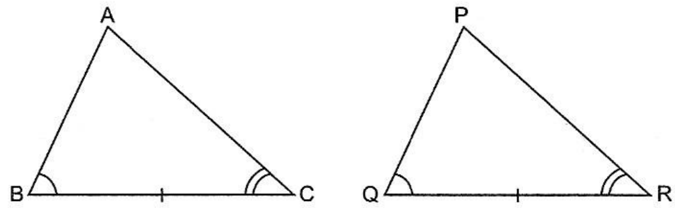
$\angle B = \angle Q, \angle C = \angle R$

and $BC = QR$

To prove: $\Delta ABC \cong \Delta PQR$

Proof: Three cases arises.

Case I. When $AB = PQ$, $\angle B = \angle Q$ and $BC = QR$



In ΔABC and ΔPQR,

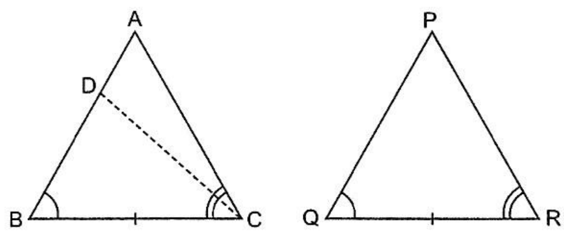
$AB = PQ$ (Assumed)

$\angle B = \angle Q$ (Given)

$BC = QR$ (Given)

$\Rightarrow$ $\Delta ABC \cong \Delta PQR$ (SAS congruence rule)

Case II. When $AB > PQ$,



Let us consider a point D on AB such that $DB = PQ$

Now, consider ΔDBC and ΔPQR $DB = PQ$ (By construction)

$\angle B = \angle Q$ (Given)

$BC = QR$ (Given)

$\Rightarrow$ $\Delta DBC \cong \Delta PQR$ (SAS congruence rule)

$\Rightarrow$ $\angle DCB = \angle PRQ$ (CPCT)

But, we are given that $\angle ACB = \angle PRQ$

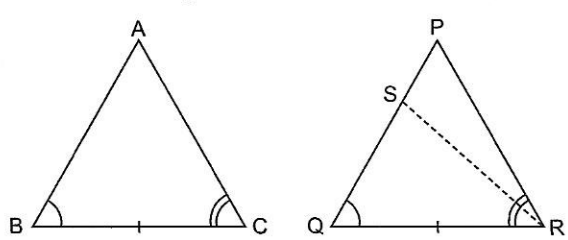
So, $\angle ACB = \angle DCB$

This is possible only when D coincides with A

i.e. $BA = QP$

So, $\Delta ABC \cong \Delta PQR$ (SAS congruence rule)

Case III. When $AB < PQ$, let us consider a point S on PQ such that $SQ = AB$ as shown in figure.



Now, in ΔABC and ΔSQR, $AB = SQ$ (By construction)

$\angle B = \angle Q$

$BC = QR$

So, $\Delta ABC \cong \Delta SQR$ (SAS congruence rule)

$\Rightarrow$ $\angle ACB = \angle SRQ$ (CPCT)

But we are given that $\angle ACB = \angle PRQ$ (As $\Delta ABC \cong \Delta PQR$)

$\Rightarrow$ $\angle SRQ = \angle PRQ$

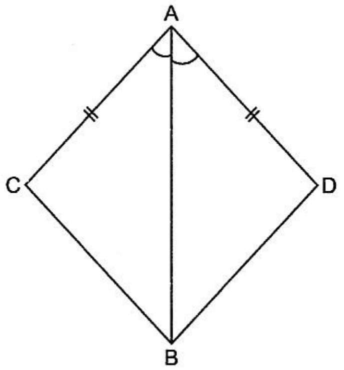
This is possible only when S coincide with P

or $QS = QP$

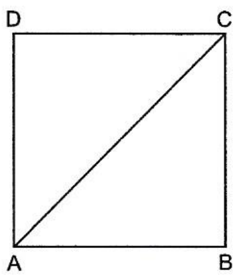
So, $\Delta ABC \cong \Delta PQR$ Hence proved.

PRACTICE QUESTIONS BASED ON EXERCISE 7.1

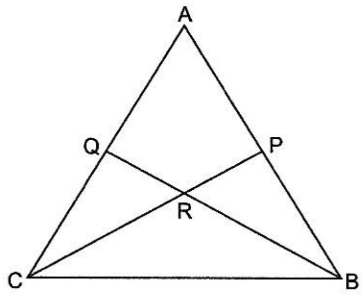
1. State the congruence in the symbolic form of two Δs PQR and DEF, if $PR = EF$, $QR = DE$ and $PQ = FD$.
2. In ΔABC and ΔDEF, $AB = DE$, $BC = EF$ and $\angle A = \angle D$. State whether the triangles are congruent or not. If yes, by which congruence rule?
3. In the given figure, state the congruence rule used in proving $\Delta ACB \cong \Delta ADB$ **[CBSE 2010]**



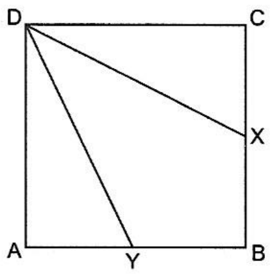
4. In ΔABC and ΔDEF, $AB = FD$, $\angle A = \angle D$. State the necessary condition according to which two triangles will be congruent by SAS congruence rule. **[CBSE 2011]**
5. In the given figure, the diagonal AC of quadrilateral ABCD bisects $\angle BAD$ and $\angle BCD$. Prove that $BC = CD$.



6. It is given that $\Delta ABC \cong \Delta FDE$ and $AB = 5$ cm. $\angle B = 30°$ and $\angle A = 80°$. Find $\angle E$ and length of side DF.
7. In the given figure, $AQ = AP$ and $BP = CQ$. Prove that $\Delta AQB \cong \Delta APC$.



8. Prove that the median of an equilateral triangle are equal.
9. BC is a line segment and line m is its perpendicular bisector. If a point P lies on m, show that P is equidistant from B and C. **[CBSE 2016]**
10. ABCD is a square and $BX = BY$. Prove that



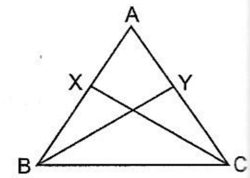
(*i*) $\Delta DCX \cong \Delta DAY$

(*ii*) $DY = DX$

(*iii*) $\angle DXC = \angle DYA$

[CBSE 2016]

6. In the figure below, ABC is a triangle in which AB = AC. X and Y are points on AB and AC such that AX = AY. Prove that ΔABY ≅ ΔACX. **[CBSE 2011]**



Sol. **Given:** In ΔABC, AB = AC and AX = AY

To prove: ΔABY ≅ ΔACX

Proof: In ΔABY and ΔACX,

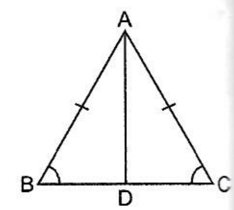
AB = AC (Given)

∠A = ∠A (Common)

AX = AY (Given)

⇒ ΔABY ≅ ΔACX (SAS congruence rule)

7. In the given figure, ΔABC is an isosceles triangle with AB = AC. If the altitude is drawn from one of its vertex, then prove that it bisects the opposite side.



Sol. **Given:** (*i*) ΔABC is an isosceles triangle with AB = AC

(*ii*) AD is the altitude drawn from vertex A on side BC.

To prove: D is mid-point of BC, i.e. BD = CD

Proof: In ΔABD and ΔACD

AB = AC (Given)

∠B = ∠C (Angles opposite to equal sides are equal)

∠ADB = ∠ADC = 90° (Given)

⇒ ΔABD ≅ ΔACD (AAS congruence rule)

⇒ BD = CD (CPCT)

Therefore, AD bisect BC. Hence proved.

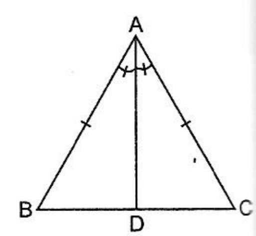
Short Answer Type Questions II [3 Marks]

8. Prove that angles opposite to equal sides of an isosceles triangle are equal. **[CBSE 2011, 2014]**

Sol. **Given:** ΔABC is an isosceles triangle with AB = AC

To prove: ∠B = ∠C

Construction: Draw AD bisector of ∠A which intersects BC at D.



Proof: In ΔBAD and ΔCAD,

AB = AC (Given)

∠BAD = ∠CAD (By construction)

AD = AD (Common)

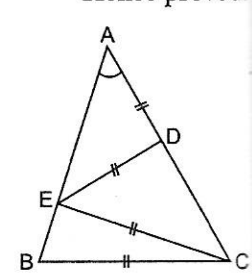
So, ΔBAD ≅ ΔCAD (SAS congruence rule)

⇒ ∠ABD = ∠ACD (CPCT)

So, ∠B = ∠C

Hence proved.

9. In the given figure, AB = AC. D is point on AC and E on AB such that AD = ED = EC = BC. Prove that ∠A : ∠B = 1 : 3 **[CBSE 2015; HOTS]**



Sol. **Given:** (*i*) AB = AC

(*ii*) AD = ED = EC = BC

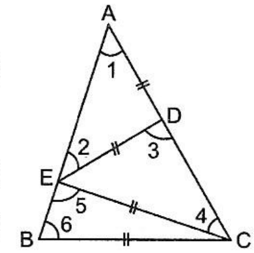
To prove: ∠A : ∠B = 1 : 3

Proof: In ΔAED,

AD = ED (Given)

⇒ ∠1 = ∠2 (Angles opposite to equal sides are equal) ...(*i*)

Also, in ΔAED, $\angle A + \angle AED + \angle ADE = 180°$



(Angle sum property of a triangle)

$\Rightarrow \quad \angle 1 + \angle 2 + \angle ADE = 180°$

$\Rightarrow \quad \angle ADE = 180° - 2\angle 1 \quad (\angle 1 = \angle 2)$

But $\quad \angle ADE + \angle CDE = 180° \quad$ (Linear pair axiom)

$\Rightarrow \quad 180° - 2\angle 1 + \angle 3 = 180°$

$\Rightarrow \quad \angle 3 = 2\angle 1 \quad ...(ii)$

Now, in ΔCDE, $\quad \angle 3 + \angle CED + \angle 4 = 180° \quad$ (Angle sum properly of a triangle)

$\Rightarrow \quad \angle CED = 180° - \angle 3 - \angle 4$

$\angle CED = 180° - 2\angle 3 \quad (\because ED = EC; \angle 3 = \angle 4) \quad ...(iii)$

Again, $\quad \angle AED + \angle CED + \angle BEC = 180° \quad$ (Linear pair axiom)

$\Rightarrow \quad \angle 2 + 180° - 2\angle 3 + \angle 5 = 180°$

$\Rightarrow \quad 2\angle 3 = \angle 2 + \angle 5 \quad ...(iv)$

In ΔBEC, $\quad EC = BC$

$\Rightarrow \quad \angle 6 = \angle 5 \quad$ (Angles opposite to equal sides are equal) ...(v)

From (i), (iv) and (v), we get

$2\angle 3 = \angle 1 + \angle 6 \quad (\angle 2 = \angle 1 \text{ and } \angle 5 = \angle 6)$

$\Rightarrow \quad 2(2\angle 1) = \angle 1 + \angle 6 \quad$ [From (ii)]

$\Rightarrow \quad 4\angle 1 = \angle 1 + \angle 6$

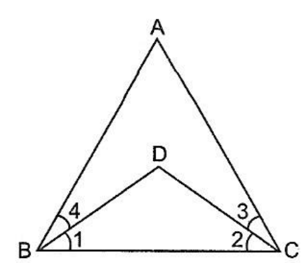
$\Rightarrow \quad \angle 6 = 3\angle 1$

$\Rightarrow \quad \angle B = 3\angle A$

$\therefore \quad \frac{\angle A}{\angle B} = \frac{1}{3}$

$\Rightarrow \quad \angle A : \angle B = 1 : 3 \quad$ Hence proved

10. In the given figure, we have $\angle ABC = \angle ACB$ and $\angle 3 = \angle 4$. Show that



(i) $\angle 1 = \angle 2$

(ii) Justify which two sides of ΔABC are equal.

Sol. (i) Given $\quad \angle ABC = \angle ACB$

$\Rightarrow \quad \angle 1 + \angle 4 = \angle 2 + \angle 3$

But $\quad \angle 3 = \angle 4 \quad$ (Given)

$\Rightarrow \quad \angle 1 = \angle 2 \quad$ Hence proved

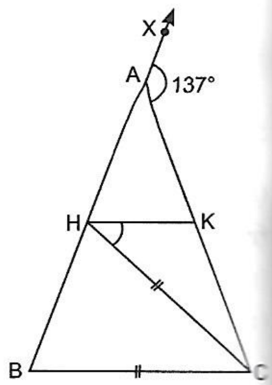
(ii) $\quad \angle ABC = \angle ACB \quad$ (Given)

$\Rightarrow \quad AC = AB$

Because in an isosceles triangle, the sides opposite to equal angles are equal.

Long Answer Type Questions [4 Marks]

11. In the given figure, AB = AC, CH = CB and HK||BC. If ∠CAX = 137°, then find ∠CHK. **[CBSE 2016; HOTS]**



Sol. Given: In ΔABC,

(*i*) AB = AC

(*ii*) CH = CB

(*iii*) HK||BC

(*iv*) ∠CAX = 137°

To find: ∠CHK

Finding: In ΔABC, AB = AC (Given)

⇒ ∠ABC = ∠ACB (Angles opposite to equal sides are equal)

But ∠CAX = ∠ABC + ∠ACB (By exterior angle theorem)

⇒ 137° = 2∠ABC (∵ ∠ACB = ∠ABC)

⇒ $\angle ABC = \frac{137°}{2} = 68.5°$

⇒ ∠ACB = 68.5°

Now, CH = CB

⇒ ∠CBH = ∠CHB (Angles opposite to equal sides are equal)

⇒ ∠CHB = 68.5° (∠CBH = ∠ABC)

Again, HK||BC (Given)

and CH is transversal

⇒ ∠BHK + ∠CBH = 180° (Co-interior angles)

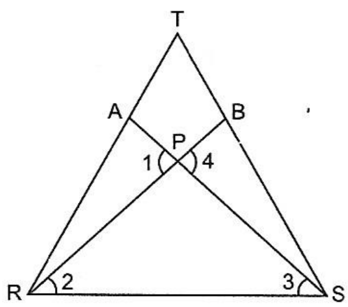
⇒ ∠CHB + ∠CHK + ∠CBH = 180° (∵ ∠BHK = ∠CHB + ∠CHK)

2∠CHB + ∠CHK = 180° (∠CBH = ∠CHB)

⇒ 2 × 68.5° + ∠CHK = 180°

⇒ ∠CHK = 180° – 137° = 43°

12. In the given figure, it is given that RT = TS, ∠1 = 2∠2 and ∠4 = 2∠3. Prove that ΔRBT ≅ ΔSAT **[CBSE 2013; HOTS]**



Sol. Given: (*i*) RT = TS

(*ii*) ∠1 = 2∠2

(*iii*) ∠4 = 2∠3

To prove: ΔRBT ≅ ΔSAT

Proof: In ΔTRS,

RT = TS (Given)

⇒ ∠TRS = ∠TSR (Angles opposite to equal sides are equal) ...(*i*)

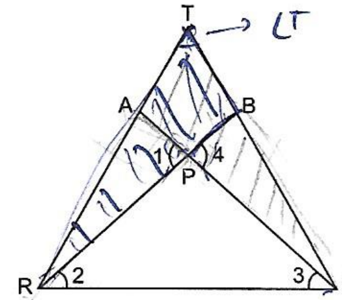
Now, SA and RB intersect at a point. Let it be P.

So, ∠1 = ∠4 (Vertically opposite angles)

⇒ 2∠2 = 2∠3

⇒ ∠2 = ∠3 ...(*ii*)

Now, in ΔRPS,



$\angle 2 = \angle 3$ (Proved above)

$\Rightarrow$ $SP = RP$ (Sides opposite to equal angles are equal) ...(*iii*)

Again from (*i*),

$\angle TRS = \angle TSR$

$\Rightarrow$ $\angle ARP + \angle 2 = \angle BSP + \angle 3$

$\Rightarrow$ $\angle ARP = \angle BSP$ (As $\angle 2 = \angle 3$) ...(*iv*)

Now, in ΔARP and ΔBSP, $\angle ARP = \angle BSP$ (From (*iv*))

$RP = SP$ (From (*iii*))

$\angle 1 = \angle 4$ (Vertically opposite angles)

$\Rightarrow$ $\Delta ARP \cong \Delta BSP$ (ASA congruence rule)

$\Rightarrow$ $AR = BS$ (CPCT)

But $RT = TS$ (Given)

$\Rightarrow$ $RT - AR = TS - BS$

$\Rightarrow$ $AT = BT$ ...(*v*)

Now, in ΔRBT and ΔSAT $RT = ST$ (Given)

$\angle T = \angle T$ (Common)

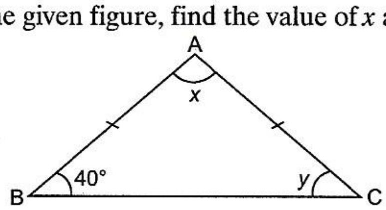
$BT = AT$ (From (*v*))

$\Rightarrow$ $\Delta RBT \cong \Delta SAT$ (SAS congruence rule)

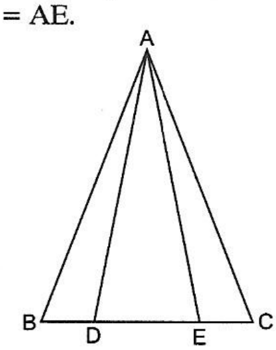
Hence proved.

PRACTICE QUESTIONS BASED ON EXERCISE 7.2

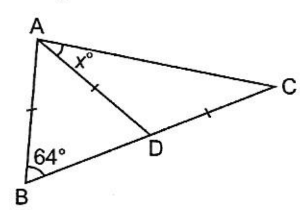
1. In an isosceles triangle ABC, $AB = AC$, $\angle A = 80°$. Find the value of $\angle B$ and $\angle C$.
2. In ΔPQR, $PQ = PR$ and $\angle Q = 80°$. Find $\angle P$.
3. In the given figure, find the value of x and y.



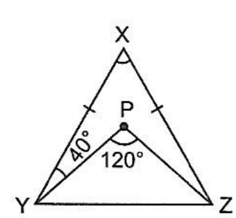
4. In the given figure, $AB = AC$ and $BE = CD$. Prove that $AD = AE$.



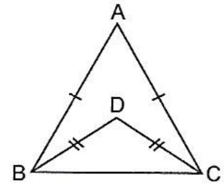
5. The base angle of an isosceles triangle is 20 more than thrice the vertex angle. Find the value of equal angles.
6. The average of two non-congruent angles of an isosceles triangle is 65°. Find the value of these two non-congruent angles **[HOTS]**
7. ΔBAD and ΔADC are two isosceles triangles. If $\angle ABD = 64°$, Find $\angle DAC$.



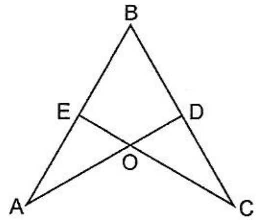
8. In ΔABC, E and D are the two points on BC such that $CE = DB$ and $\angle AEC = \angle ADB$. Prove that
   (*i*) ΔAED is an isosceles triangle.
   (*ii*) $\Delta CAD \cong \Delta BAE$
9. In the given figure, ΔXYZ and ΔPYZ are two isosceles triangle on the same base YZ with $XY = XZ$ and $PY = PZ$. If $\angle P = 120°$ and $\angle XYP = 40°$, then find $\angle YXZ$. **[CBSE 2010]**



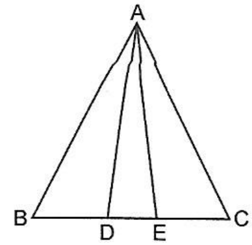
10. In the given figure, AB = AC and DB = DC. Find the ratio between the ∠ABD and ∠ACD.



11. In the given figure, ∠A = ∠C and AB = BC. Prove that ΔABD ≅ ΔCBE.



12. In the given figure, AD = AE and D and E are the points on BC such that BD = EC. Prove that AB = AC.



Some More Criteria for Congruence of Triangles

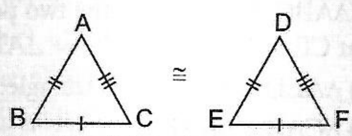
Two More Criteria for Congruence of Triangles

SSS (Side-Side-Side) Congruence Rule

Equal corresponding sides of two triangles

In ΔABC and ΔDEF,
- AB = DE
- AC = DF
- BC = EF

ΔABC ≅ ΔDEF



ΔABC ≅ ΔDEF

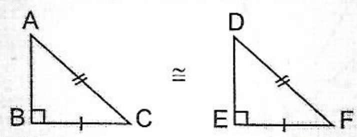
SSS Congruence rule: If three sides of one triangle are equal to three sides of another triangle, then the two triangles are congruent.

RHS (Right angle-Hypotenuse Side) Congruence Rule

In two right-angled triangles corresponding side and hypotenuse.

In ΔABC and ΔDEF with ∠B = ∠E = 90°,
- BC = EF
- AC = DF

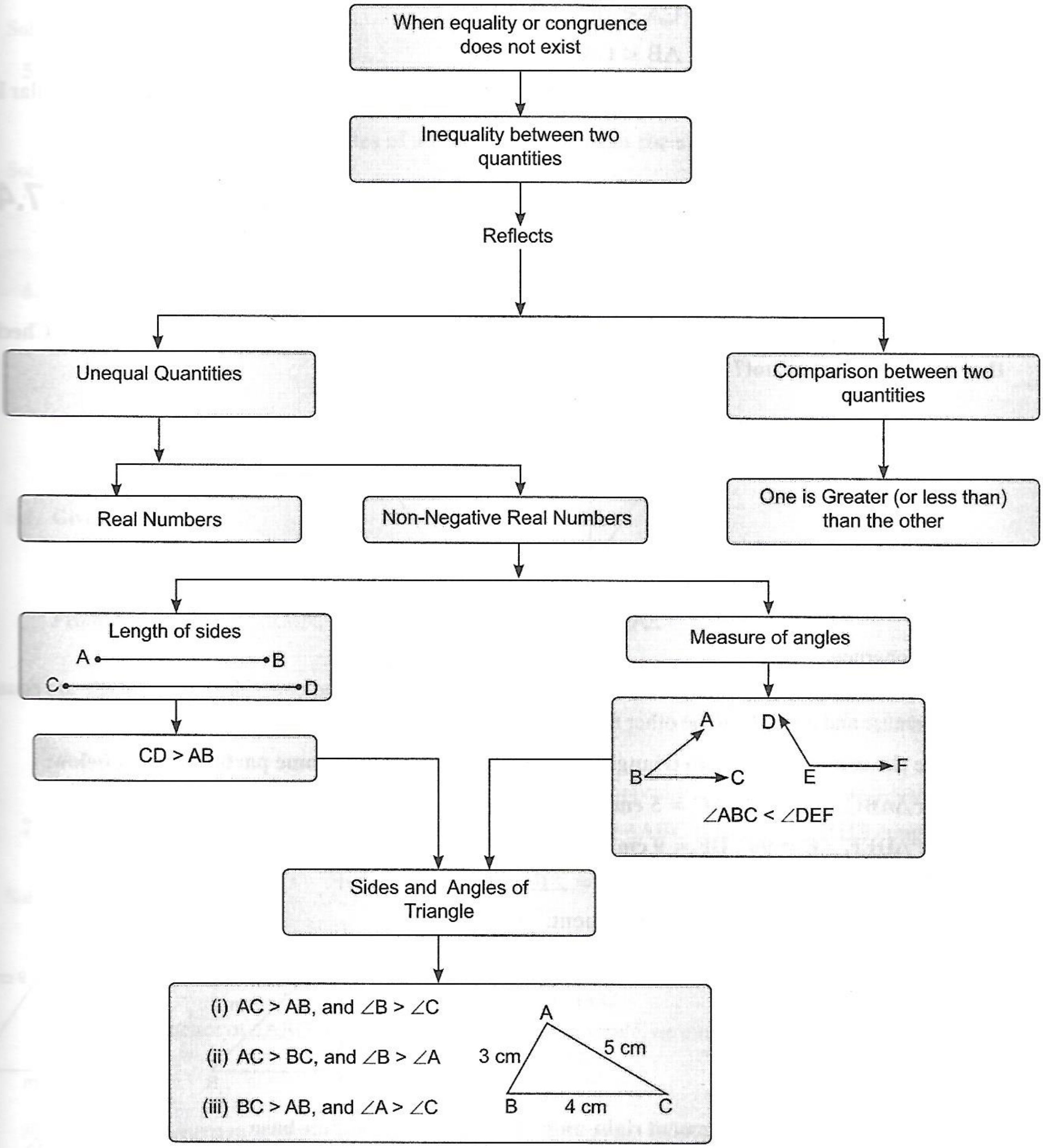
ΔABC ≅ ΔDEF



ΔABC ≅ ΔDEF

RHS Congruence rule: If in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.

Inequalities in a Triangle



(*i*) If two sides of a triangle are unequal, the angle opposite to the longer side is larger (or greater)

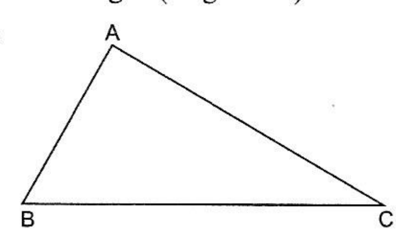
(*ii*) In any triangle, the side opposite to the larger (greater) angle is longer.

(*iii*) The sum of any two sides of a triangle is greater than the third side

(*a*) AB + BC > CA

(*b*) BC + CA > AB

(*c*) CA + AB > BC



This gives us

(*a*) AB > CA – BC, i.e. CA –BC < AB

(*b*) BC > AB – CA, i.e. AB – CA < BC

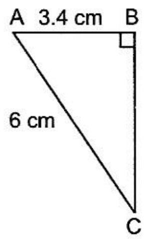
(*c*) CA > BC – AB, i.e. BC – AB < CA.

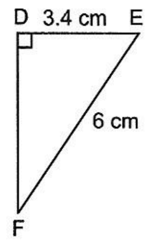
(*iv*) Of all line segments that can be drawn to a given line from a point not lying on it, the perpendicular line segment is the shortest.

SOLVED QUESTIONS BASED ON EXERCISES 7.3 AND 7.4

Very Short Answer Type Questions [1 Mark]

1. In two right-angled ΔABC and ΔDEF, the measurement of hypotenuse and one side is given. Check if they are congruent or not? If yes, state the rule.





Sol. Yes, ΔABC ≅ ΔEDF

By RHS congruence rule.

RHS Congruence rule: If in two right angled triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.

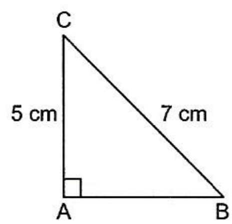
2. Examine the congruence of two triangles, whose measurements of some parts are given below:

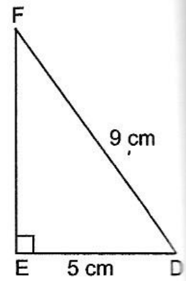
(*i*) for ΔABC, ∠A = 90°, AC = 5 cm, BC = 7 cm

(*ii*) for ΔDEF, ∠E = 90°, DF = 9 cm, DE = 5 cm

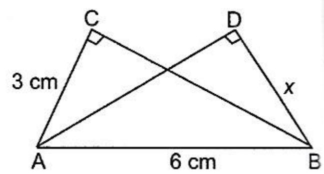
Sol. From the figure, AC = DE = 5 cm. ∠A = ∠E = 90° but BC ≠ DF.

Hence, the given triangles are not congruent.





3. ΔACB and ΔADB are two congruent right-angled triangles on the same base AB (= 6 cm) as shown in figure. If AC = 3 cm, find BD.



Sol.	ΔABC ≅ ΔABD	(RHS congruence rule given)
⇒	AC = BD	(CPCT)
⇒	BD = 3cm	(∵ AC = 3 cm, given)

4. **Fill in the blanks:**

   (*i*) **If two angles of a triangle are unequal then the smaller angle has the _____ side opposite to it.**

   (*ii*) **The sum of any two sides of a triangle is _______ than the third side.**

Sol. (*i*) smaller (*ii*) greater

5. **Which of the following statements are true and which are false?**

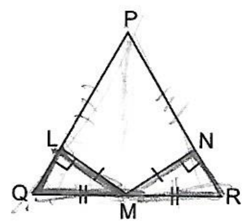
   (*i*) **If two sides of a triangle are unequal, then longer side has the smaller angle opposite to it.**

   (*ii*) **The sum of the three sides of a triangle is less than the sum of its three altitudes.**

Sol. (*i*) false (*ii*) false

Short Answer Type Questions I [2 Marks]

6. **In the given figure, LM = MN, QM = MR, ML ⊥ PQ and MN ⊥ PR. Prove that PQ = PR.**



Sol. Given: LM = MN, QM = MR

ML ⊥ PQ and MN ⊥ PR

To prove: PQ = PR

Proof: In ΔQML and ΔRMN,

LM = MN (Given)

∠L = ∠N (Each 90°)

QM = MR (Given)

⇒ ΔQML ≅ ΔRMN (RHS congruence rule)

⇒ ∠LQM ≅ ∠NRM (CPCT)

⇒ PQ = PR (Sides opposite to equal angles are equal) Hence proved.

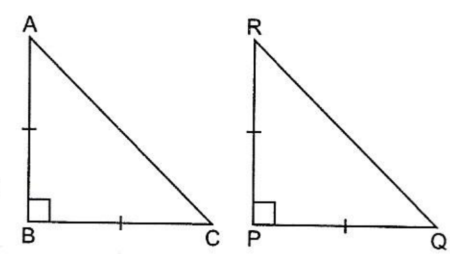
7. **What additional information is needed for establishing ΔABC ≅ ΔRPQ, by RHS congruence rule, if it is given that AB = RP and ∠B = ∠P = 90°?**

Sol. Given ΔABC ≅ ΔRPQ

AB = RP

∠B = ∠P = 90°

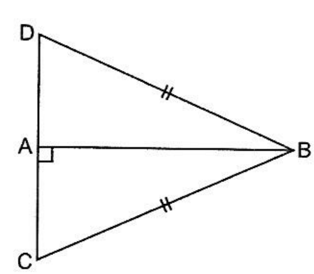
⇒ A ↔ R, B ↔ P and C ↔ Q



So, for congruence of ΔABC and ΔRPQ by RHS congruence rule, we must have

AC = RQ

8. **Write the congruence statement by the information shown in the figure.**



Sol. From the figure,

In ΔBAC and ΔBAD,

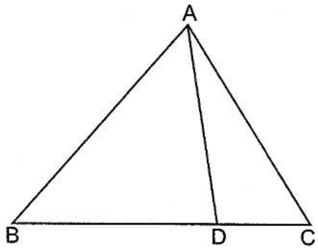
AB = AB (Common)

∠BAC = ∠BAD (Each 90°)

BC = BD (Given)

⇒ ΔBAC ≅ ΔBAD (RHS congruence rule)

9. In the given figure, AB > AC and D is any point on side BC of ΔABC. Prove that AB > AD.



Sol. AB > AC (Given)

∴ ∠C > ∠B (Angle opposite to longer side is larger) ...(*i*)

Now, ∠ADB is the exterior angle of ΔADC

⇒ ∠ADB = ∠DAC + ∠C

⇒ ∠ADB > ∠C ...(*ii*)

Therefore, from (*i*), we get

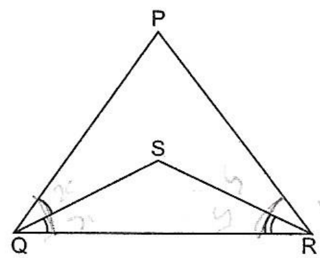
⇒ ∠ADB > ∠B

Now in ΔABD;

∠ADB > ∠B

⇒ AB > AD (Side opposite to greater angle is longer) Hence proved.

10. In the given figure, PQ > PR, QS and RS are the bisectors of ∠Q and ∠R respectively. Prove that SQ > SR.



Sol. Proof: In ΔPQR,

PQ > PR (Given)

⇒ ∠PRQ > ∠PQR (Angle opposite to longer side is larger)

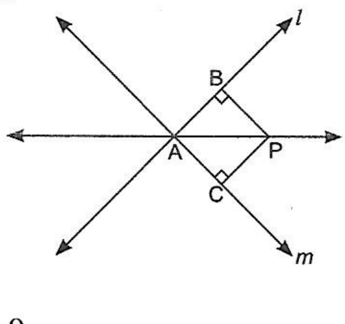
⇒ $\frac{1}{2}\angle PRQ > \frac{1}{2}\angle PQR$

⇒ ∠SRQ > ∠SQR (Given QS and RS are the bisectors of ∠Q and ∠R respectively)

⇒ SQ > SR (Side opposite to greater angle is larger) Hence proved.

Short Answer Type Questions II [3 Marks]

11. P is a point equidistant from two lines *l* and *m* intersecting at point A as shown in figure. Show that line AP bisects the angle between them.



Sol. **Given:** (*i*) Lines l and m intersect each other at point A.

(*ii*) From figure, $PB \perp l$, $PC \perp m$

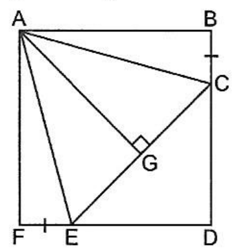
(*iii*) PB = PC

To prove: Line AP bisects $\angle BAC$.

Proof: In ΔPAB and ΔPAC,

	$PB = PC$	(Given)
	$\angle PBA = \angle PCA = 90°$	(Given)
	$PA = PA$	(Common)
⇒	$\Delta PAB \cong \Delta PAC$	(RHS congruence rule)
⇒	$\angle PAB = \angle PAC$	(CPCT)
⇒ Line AP bisects $\angle BAC$.		Hence proved.

12. ABDF is a square and BC = EF in the given figure. Prove that



(*i*) $\Delta ABC \cong \Delta AFE$ (*ii*) $\Delta ACG \cong \Delta AEG$ **[HOTS]**

Sol. **Given:** (*i*) ABDF is square

(*ii*) BC = EF

To prove: (*i*) $\Delta ABC \cong \Delta AFE$

(*ii*) $\Delta ACG \cong \Delta AEG$

Proof:

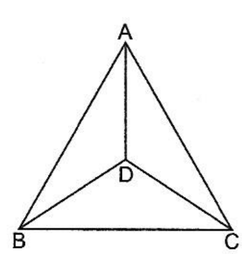
(*i*) In ΔABC and ΔAFE,

	$AB = AF$	(All sides of a square are equal)
	$BC = FE$	(Given)
and	$\angle ABC = \angle AFE = 90°$	(Each angle of a square is a right angle)
⇒	$\Delta ABC \cong \Delta AFE$	(SAS congruence rule)
⇒	$AC = AE$	(CPCT)
		Hence proved.

(*ii*) In ΔACG and ΔAEG,

	$AC = AE$	(Proved above)
	$AG = AG$	(Common)
	$\angle AGC = \angle AGE = 90°$	(Given)
⇒	$\Delta ACG \cong \Delta AEG$	(RHS congruence rule) Hence proved.

13. In the given figure, AB = AC and D is a point in the interior of ΔABC such that $\angle DBC = \angle DCB$. Prove that AD bisects $\angle BAC$ of ΔABC.



Sol. In ΔBDC,

$\angle DBC = \angle DCB$ (Given)

$\Rightarrow$ $BD = CD$ (Sides opposite to equal angles are equal)

Now, in ΔABD and ΔACD,

$AB = AC$ (Given)

$BD = CD$ (Proved above)

and $AD = AD$ (Common)

$\Rightarrow$ $\Delta ABD \cong \Delta ACD$ (SSS congruence rule)

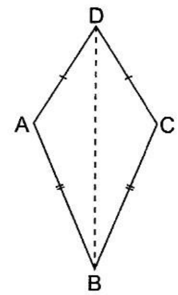
$\therefore$ $\angle BAD = \angle CAD$ (CPCT)

Hence, AD bisects $\angle BAC$. Hence Proved.

14. In the given figure, AD = CD and AB = CB. Prove that

(*i*) $\Delta ABD \cong \Delta CBD$

(*ii*) BD bisects $\angle ABC$.



Sol. Given: AD = CD and AB = CB

To prove: (*i*) $\Delta ABD \cong \Delta CBD$

(*ii*) $\angle ABD = \angle CBD$, i.e. BD bisects $\angle ABC$.

Proof: (*i*) In ΔABD and ΔCBD,

$AB = CB$ (Given)

$AD = CD$ (Given)

$BD = BD$ (Common)

$\Rightarrow$ $\Delta ABD \cong \Delta CBD$ (SSS congruence rule)

(*ii*) Since $\Delta ABD \cong \Delta CBD$ (Proved above)

$\Rightarrow$ $\angle ABD = \angle CBD$ (CPCT)

$\Rightarrow$ BD bisects $\angle ABC$. Hence proved.

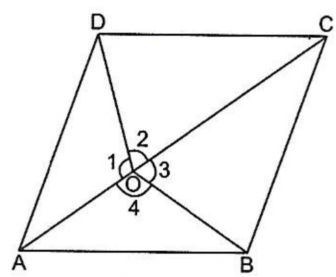
15. A point O is taken inside an equilateral four sided figure ABCD such that its distances from the angular points D and B are equal. Show that AO and OC are together form one and the same straight line.

[HOTS]

Sol. Given: O is a point anywhere inside an equilateral four sided figure ABCD such that OD = OB.

To prove: AO and OC are in the same straight line.

Proof: In ΔAOD and ΔBOA,



$AD = AB$ (Given sides of ABCD are equal)

$AO = AO$ (Common)

$OD = OB$ (Given)

$\Rightarrow$ $\Delta AOD \cong \Delta BOA$ (SSS congruence rule)

$\Rightarrow$ $\angle 1 = \angle 4$ (CPCT)

Similarly, in ΔCOD and ΔCOB,

$CO = CO$ (Common)

$CD = CB$ (Given sides of ABCD are equal)

$OD = OB$ (Given)

$\Rightarrow$ $\Delta COD \cong \Delta COB$ (SSS congruence rule)

$\Rightarrow$ $\angle 2 = \angle 3$ (CPCT)

But $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360°$ (Complete angle)

$\Rightarrow$ $\angle 1 + \angle 2 + \angle 2 + \angle 1 = 360°$ ($\because$ $\angle 4 = \angle 1$ and $\angle 3 = \angle 2$)

$\Rightarrow$ $2(\angle 1 + \angle 2) = 360°$

$\Rightarrow$ $\angle 1 + \angle 2 = 180°$

$\Rightarrow$ $\angle AOD + \angle COD = 180°$

But these are the linear pair angles formed by a line OD stands on AOC.

Therefore, AO and OC are together form one and the same straight line.

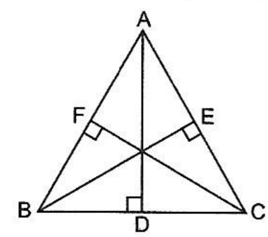
$\Rightarrow$ AOC is a straight line. Hence proved.

Long Answer Type Questions [4 Marks]

16. Prove that the sum of three altitudes of a triangle is less than the sum of the three sides of a triangle.

[HOTS]

Sol. Given: In ΔABC, AD, BE and CF are the altitudes on sides BC, CA and AB respectively.



To prove: $AD + BE + CF < AB + BC + CA$

Proof: Since perpendicular line segment is the shortest line segment, then when $AD \perp BC$ we have

$AB > AD$ and $AC > AD$

$\Rightarrow$ $AB + AC > AD + AD$

$\Rightarrow$ $AB + AC > 2AD$ ...(*i*)

Similarly, when $BE \perp AC$, then $BA + BC > 2BE$ ...(*ii*)

and, when $CF \perp AB$ $CA + CB > 2CF$ ...(*iii*)

Adding (*i*), (*ii*) and (*iii*), we get

$AB + AC + BA + BC + CA + CB > 2AD + 2BE + 2CF$

$\Rightarrow$ $2AB + 2BC + 2CA > 2AD + 2BE + 2CF$

$\Rightarrow$ $2(AB + BC + CA) > 2(AD + BE + CF)$

$\Rightarrow$ $AB + BC + CA > AD + BE + CF$

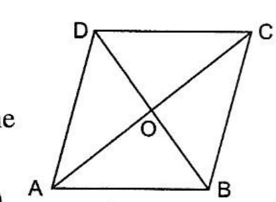
$AD + BE + CF < AB + BC + CA$ Hence proved.

17. Diagonal AC and BD of quadrilateral ABCD intersects each other at O. Prove that

(*i*) $AB + BC + CD + DA > AC + BD$

(*ii*) $AB + BC + CD + DA < 2(AC + BD)$

Sol. Given: AC and BD are the diagonals of quadrilateral ABCD.



(*i*) **To prove:** $AB + BC + CD + DA > AC + BD$

Proof: We know that the sum of any two sides of a triangle is always greater than the third side. Therefore,

In ΔABC, $AB + BC > AC$ ...(*i*)

In ΔBCD, $BC + CD > BD$ ...(*ii*)

In ΔCDA, $CD + DA > CA$ ...(*iii*)

In ΔABD, $AB + AD > BD$ ...(*iv*)

Adding (*i*), (*ii*), (*iii*) and (*iv*), we get

$2(AB + BC + CD + DA) > 2(AC + BD)$

$\Rightarrow$ $AB + BC + CD + DA > AC + BD$ Hence proved.

(*ii*) **To prove:** $AB + BC + CD + DA < 2(AC + BD)$

Proof: In ΔOAB, $OA + OB > AB$ ...(*i*)

In ΔBOC, $OB + OC > BC$ ...(*ii*)

In ΔCOD, $OC + OD > CD$ ...(*iii*)

In ΔAOD, $OA + OD > DA$ ...(*iv*)

Adding (*i*), (*ii*), (*iii*) and (*iv*), we get

$$2(OA + OB + OC + OD) > AB + BC + CD + DA$$

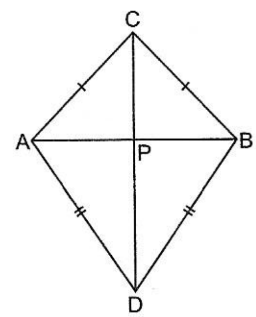
$$2[(OA + OC) + (OB + OD)] > AB + BC + CD + DA$$

$$2(AC + BD) > AB + BC + CD + DA$$

$$AB + BC + CD + DA < 2(AC + BD)$$

Hence proved.

18. AB is a line segment C and D are points on opposite sides of AB such that each of them is equidistant from the point A and B as shown in figure. Show that the line CD is the perpendicular bisector of AB.



Sol. Given: $CA = CB$ and $DA = DB$

To prove: (*i*) $CD \perp AB$

(*ii*) CD bisects AB

Proof: Let CD intersects AB at P.

Consider ΔCAD and ΔCBD,

$CA = CB$ (Given)

$DA = DB$ (Given)

$CD = CD$ (Common)

$\Rightarrow$ $\Delta CAD \cong \Delta CBD$ (SSS congruence rule)

$\Rightarrow$ $\angle ACD = \angle BCD$ (CPCT)

Again, in ΔCAP and ΔCBP,

$CA = CB$ (Given)

$CP = CP$ (Common)

$\angle ACP = \angle BCP$ ($\because \angle ACD = \angle BCD$. Proved above)

$\Rightarrow$ $\Delta CAP \cong \Delta CBP$ (SAS congruence rule)

$\Rightarrow$ $AP = BP$ (CPCT)

and $\angle APC = \angle BPC$ (CPCT)

But these are the linear pair angles.

Therefore, $\angle APC + \angle BPC = 180°$

$\Rightarrow$ $2\angle APC = 180°$

$\Rightarrow$ $\angle APC = 90°$ $\Rightarrow$ $CD \perp AB$

Hence, $AP = BP$ and $\angle APC = 90°$. This indicates that CD is perpendicular bisector of AB. Hence proved.

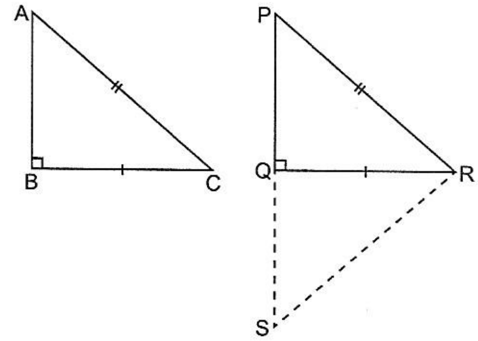
19. Prove that two right triangles are congruent, if the hypotenuse and a side of one triangle are respectively equal to the hypotenuse and a side of the other triangle. **[HOTS]**

Sol. Given: (*i*) ΔABC and ΔPQR are the two right-angled triangles with $\angle B = 90°$ and $\angle Q = 90°$.

(*ii*) $AC = PR$ and $BC = QR$

To prove: $\Delta ABC \cong \Delta PQR$

Construction: Produce PQ to S such that QS = AB. Join S and R.



Proof: In ΔABC and ΔSQR, we have

$AB = SQ$ (By construction)

$BC = QR$ (Given)

$\angle ABC = \angle SQR$ (Each 90°)

$\Rightarrow$ $\Delta ABC \cong \Delta SQR$ (SAS congruence rule)

$\Rightarrow$ $\angle A = \angle S$ (CPCT)

and $AC = SR$ (CPCT)

But $AC = PR$ (Given)

$\Rightarrow$ $SR = PR$

$\Rightarrow$ $\angle P = \angle S$ (Angles opposite to equal sides of ΔSPR are equal)

i.e. $\angle A = \angle P$ ($\angle A = \angle S$ and $\angle S = \angle P$, Proved above)

Now, in ΔABC and ΔPQR,

$\angle A = \angle P$ (Proved above)

$\angle B = \angle Q = 90°$ (Given)

$\therefore$ $\angle C = \angle R$ (By angle sum property of a triangle)

Again, in ΔABC and ΔPQR,

$BC = QR$ (Given)

$AC = PR$ (Given)

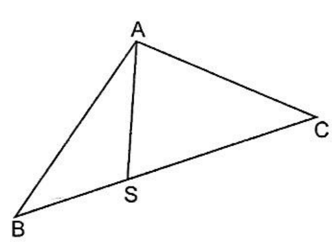
$\angle C = \angle R$ (Proved above)

$\Rightarrow$ $\Delta ABC \cong \Delta PQR$ (SAS congruence rule)

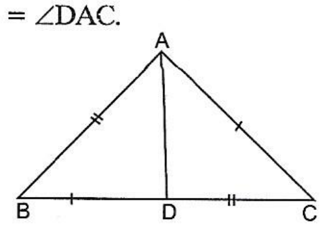
Hence proved.

PRACTICE QUESTIONS BASED ON EXERCISES 7.3 AND 7.4

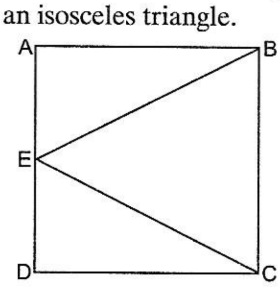
1. State the relation between the altitudes BD and CE respectively an isosceles ΔABC with $AB = AC$.
2. Is it possible to construct a triangle with length of its sides as 5 cm, 6 cm and 11 cm? Give reasons for your answer.
3. In the following figure, S is any point on side BC of ΔABC. Prove that $AB + BC + CA > 2AS$ **[CBSE 2011]**



4. In the equilateral triangle ABC, if BC is extended to D, then prove that $\angle ABD > \angle BAC$. What conclusion would you like to draw?
5. In the given figure, ΔABC and ΔACD are such that $BD = AC$, $AB = DC$. Prove that $\angle ADB = \angle DAC$. **[HOTS]**

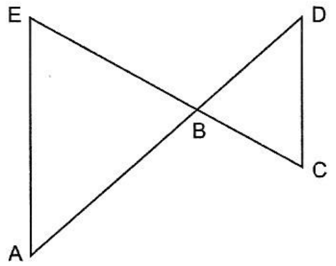


6. In the given figure, ABCD is a square. E is the mid-point of AD. BE and CE are joined. Prove that ΔBEC is an isosceles triangle.

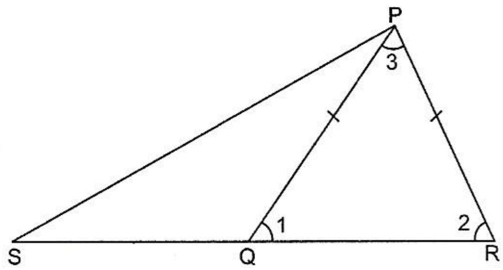


7. ΔABC is an isosceles triangle with $AB = AC$ and AD is the altitude of the triangle. Prove that AD is also a median of the triangle.

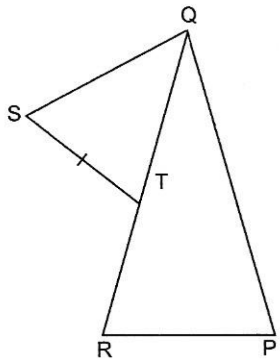
8. In the given figure, $\angle E > \angle A$ and $\angle C > \angle D$. Prove that $AD > EC$.



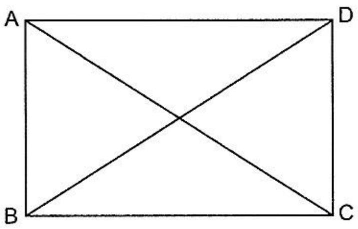
9. In the following figure, $PQ = PR$, show that $PS > PQ$.



10. In the given figure, T is a point on side QR of ΔPQR and S is a point such that $RT = ST$. Prove that: $PQ + PR > QS$.

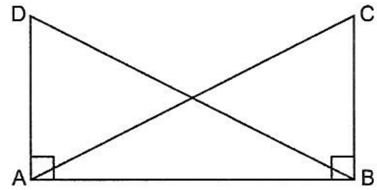


11. In the following figure, ABCD represents a rectangle in which its diagonal AC and BD are equal. Prove that $\Delta ABC \cong \Delta DCB$.



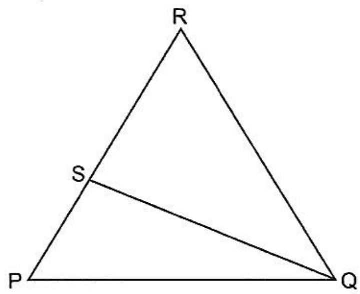
12. If the altitude drawn from the vertices of ΔABC to the opposite sides are equal, prove that the triangle is equilateral.

13. In the given figure, $DA \perp AB$, $CB \perp AB$ and $AC = BD$. Prove that $BC = AD$.



14. O is any point in the interior of ΔPQR. Prove that: $2\,(OP + OQ + OR) = PQ + QR + RP$

15. In the given figure, $PQ = PR$ and S is any point on side PR. Prove that $RS < QS$.



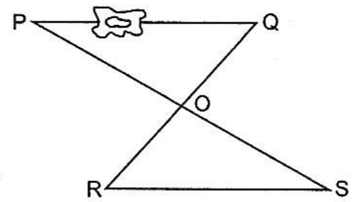
16. If D is a point on the side BC of ΔABC such that AD bisects $\angle BAC$, then prove that $BA > BD$.

17. Prove that the sum of any two sides of a triangle is greater than the third side.

18. S is any point in the interior of ΔPQR, prove that $(SQ + SR) < (PQ + PR)$. **[HOTS]**

Value Based Questions

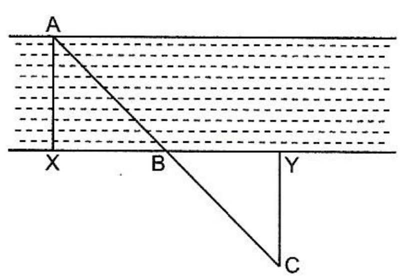
1. During education tour of class IX, the teacher asked the students to measure the distance between the two objects P and Q including an obstacle between them. This obstacle prevents the students for direct measurement. One of the students devises an ingenious solution to the problem. Firstly, she fixes a pole at a convenient point D so that both P and Q are visible. Then, she fixes the another pole at point S on the line PO produced such that $PO = SO$. In a similar way, she fixes a third pole at point R on the extended line QO such that $QO = RO$. Then she measures the distance between R and S.



(*i*) Is she able to measure the distance between P and Q? Justify.

(*ii*) What ability does this develop in students?

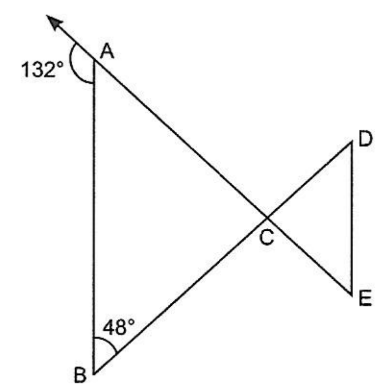
2. A student Ram wishes to find the width AB of the river without crossing it. His friend Shyam helps him to complete the task. Ram stands at point X just opposite to a fixed pole at A on the other side of the river, Shyam measures the distance between point X and B using a measuring tape. Now, he asked Ram to stand at a point Y such that XB = YB. From point Y, he went perpendicular to the bank of river and reached to a point 'C' such that A, B and C lies on the same straight line. He measures the distance YC and determines the width of the river.



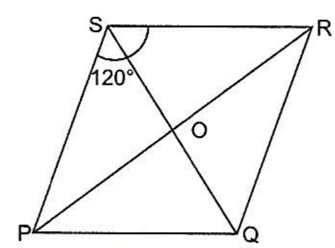
(*i*) Justify the determination of width of river by measuring the distance YC.

(*ii*) What values Shyam is exhibiting by doing so?

3. Mohan has two triangular plots connected to each other as shown in the figure. He thought to give the bigger isosceles triangular part with AC = BC to his daughter and son equally. What values he is exhibiting by doing so? How a triangle can be divided into two parts of equal area? Also, find the value of ∠DCE.



4. PQRS is a rhombus with ∠PSR = 120°. There are two fire stations at S and R. Mohan observes the fire at O as shown and decided to call the fire station.



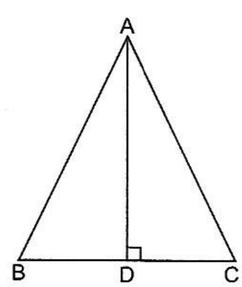
(*i*) Which fire station should he call to reach early and why?

(*ii*) What values are depicted by Mohan here?

INTEGRATED EXERCISE

Very Short Answer Type Questions [1 Mark]

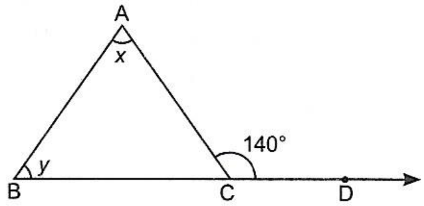
1. If ΔABC is congruent to ΔDEF by SSS congruence rule, then which of the corresponding angles of two congruent triangles are equal? **[CBSE 2011]**
2. In a ΔPQR, ∠QPR = 80° and PQ = PR. Write the value of ∠R and ∠Q. **[CBSE 2010]**
3. Given two right angled triangles ABC and PRQ, such that ∠A = 20°, ∠Q = 20° and AC = QP. Write the correspondence, if triangles are congruent. **[HOTS]**
4. In ΔABC, ∠B = ∠C and AD ⊥ BC as shown in figure. State the congruence criteria by which ΔABD ≅ ΔACD.



5. Examine whether the ΔABC and ΔPQR are congruent or not for the followings measurements.
For ΔABC, AB = 4cm, AC = 7 cm, ∠B = 90°
For ΔPQR, PR = 4 cm, QR = 7 cm, ∠P = 90°
If yes, write the result in symbolic form.

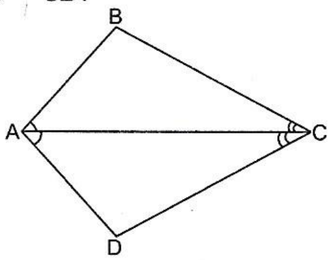
6. In ΔABC, $\angle A = 100°$, $\angle B = 30°$ and $\angle C = 50°$. Write the inequality relation between sides AB and AC. Justify it.

7. In the figure given below, find the value of $\angle x + \angle y$.

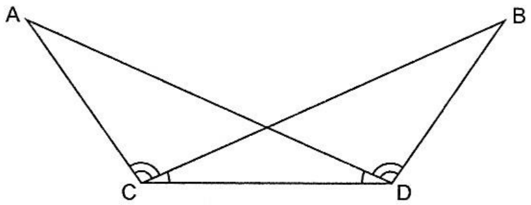


Short Answer Type Questions I [2 Marks]

8. In the given figure, diagonal AC of a quadrilateral ABCD bisects $\angle A$ and $\angle C$. Prove that AB = AD and CB = CD.

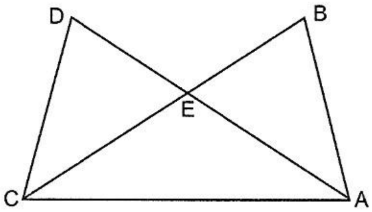


9. In the given figure, $\angle BCD = \angle ADC$ and $\angle ACB = \angle BDA$. Prove that AD = BC and $\angle A = \angle B$.



10. The vertical angle of an isosceles ΔABC is 60°. Prove that ΔABC is an equilateral triangle.

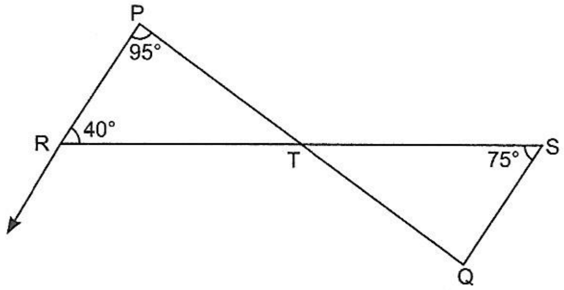
11. In the given figure, AB = CD and AD = BC. Prove that $\Delta ADC \cong \Delta CBA$.



12. In ΔABC, $2\angle A = 3\angle B = 6\angle C$. Find $\angle A$, $\angle B$ and $\angle C$.

13. In ΔPQR, $\angle P = 110°$ and $\angle R = 60°$. Which side of the triangle is the smallest? Give reasons for your answer. **[CBSE 2011]**

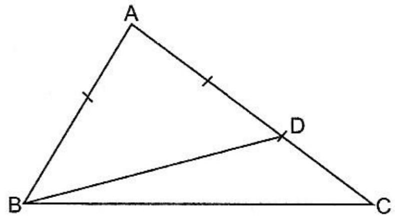
14. In the given figure, line segments PQ and RS intersect each other at a point T such that $\angle PRT = 40°$, $\angle RPT = 95°$ and $\angle TSQ = 75°$. Find $\angle SQT$. **[CBSE 2011]**



15. AD is bisector of $\angle A$ of ΔABC, where D lies on BC. Prove that AB > BD and AC > CD.

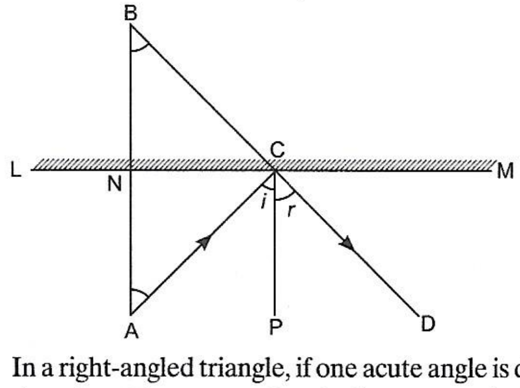
Short Answer Type Questions II [3 Marks]

16. In the given figure, AC > AB and D is a point on AC such that AB = AD. Prove that (*i*) CD < BC. (*ii*) AC – AB < BC.



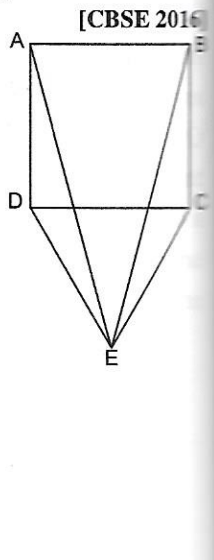
17. AD and BE are respectively altitudes of ΔABC such that AE = BD. Prove that AD = BE.

18. The image of an object placed at point A before the plane mirror LM is seen at the point B by an observer at D as shown in the figure. Prove that the image is far behind the mirror as the object is in front of the mirror.



19. In a right-angled triangle, if one acute angle is double the other. Then prove that the hypotenuse is double the smallest side. **[CBSE 2016]**

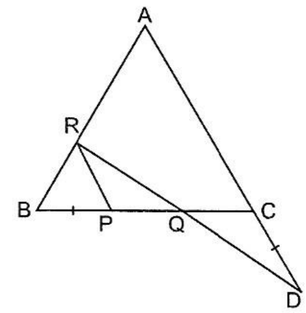
20. In the given figure, ΔCDE is an equilateral triangle formed on a side CD of a square ABCD. Show that $\Delta ADE \cong \Delta BCE$ **[NCERT Exemplar]**



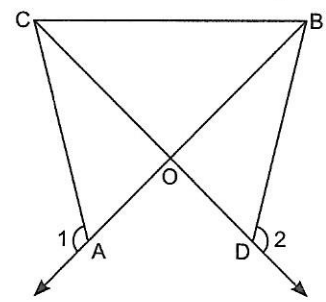
21. The bisectors of angles B and C of an isosceles ΔABC with $AB = AC$ intersect each other at O. Show that external angle adjacent to $\angle ABC$ is equal to $\angle BOC$. **[NCERT Exemplar]**

Long Answer Type Questions [4 Marks]

22. P is a point on the bisector of $\angle ABC$. The line through P parallel to BA meets BC at Q. Prove that ΔBPQ is an isosceles triangle. **[NCERT Exemplar]**
23. ΔABC is a right-angled triangle with $AB = AC$. If bisector of $\angle A$ meets BC at D, then prove that $BC = 2AD$. **[NCERT Exemplar]**
24. O is a point in the interior of a square ABCD such that ΔOAB is an equilateral triangle. Show that ΔOCD is an isosceles triangle. **[NCERT Exemplar]**
25. In a right-angled triangle, prove that the line segment joining the mid-point of the hypotenuse to the opposite vertex is half the hypotenuse. **[NCERT Exemplar]**
26. The line segment joining the mid-points M and N of parallel sides AB and DC respectively of a trapezium ABCD is perpendicular to both the sides AB and DC. Prove that $AD = BC$. **[NCERT Exemplar]**
27. ΔABC is a right-angled triangle such that $AB = AC$ and bisector of $\angle C$ intersects the side AB at D. Prove that $AC + AD = BC$. **[NCERT Exemplar]**
28. In the given figure, ΔABC is an equilateral triangle in which PR || AC. If AC is produced to D such that $CD = BP$, then prove that PC is bisected by RD at Q.



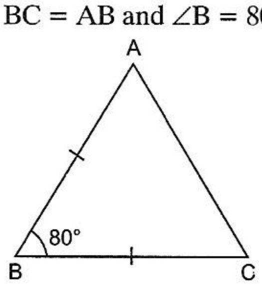
29. In the given figure, $OA = OD$ and $\angle 1 = \angle 2$. Prove that ΔOCB is an isosceles triangle.



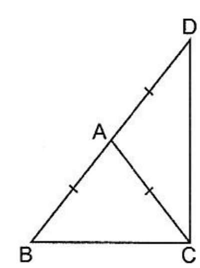
[CBSE 2015]

ASSESS YOURSELF

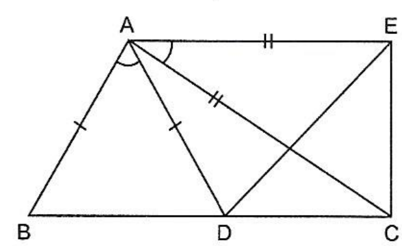
1. In ΔABC, $\angle A = 70°$ and $\angle B = 30°$. Which side of this triangle is the longest? Give reasons for your answer.
2. In ΔABC and ΔPQR, if $AB = AC$, $\angle C = \angle P$ and $\angle B = \angle Q$, then state the nature of both triangles. Check whether these triangles are congruent or not?
3. It is given that $\Delta ABC \cong \Delta RPQ$. Is it true to say that $BC = QR$? Give reasons for your answer.
4. ΔABC is an isosceles triangle with $AB = AC$. BD and CD are its two medians. Prove that these two medians are equal in length.
5. If AD is the bisector of $\angle BAC$ in ΔABC, then prove that $AB > BD$.
6. In ΔABC, $BC = AB$ and $\angle B = 80°$. Find $\angle A$.



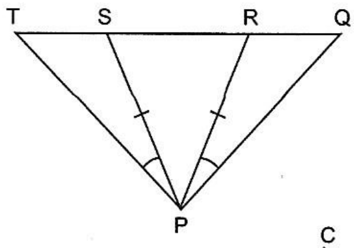
7. ΔABC is an isosceles triangle with $AB = AC$. The side BA is produced to a point D such that $AB = AD$ as shown in figure. Prove that $\angle BCD$ is a right angle.



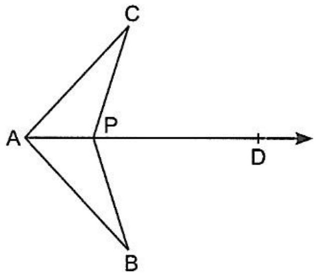
8. In the given figure, if $AB = AD$, $AC = AE$ and $\angle BAD = \angle EAC$, then prove that $BC = DE$.



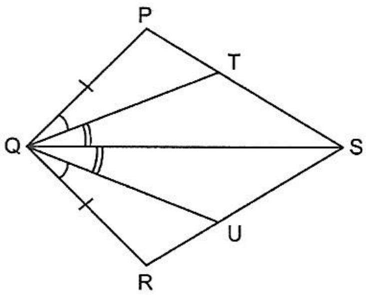
9. In the given figure, PS = PR, ∠TPS = ∠QPR. Prove that PT = PQ.



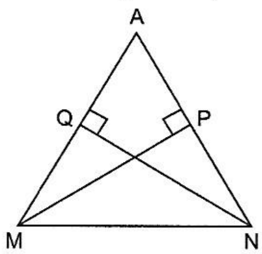
10. In the given figure, ∠CPD = ∠BPD and AD is the bisector of ∠BAC. Prove that ΔCAP ≅ ΔBAP and hence, CP = BP.



11. In the given figure, PQRS is a quadrilateral and T and U are respectively points on PS and RS such that PQ = RQ, ∠PQT = ∠RQU and ∠TQS = ∠UQS. Prove that QT = QU.



12. ΔLMN is a triangle in which altitude MP and NQ to sides LN and LM respectively are equal. Show that

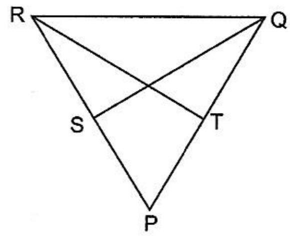


(*i*) ΔLMP ≅ ΔLNQ

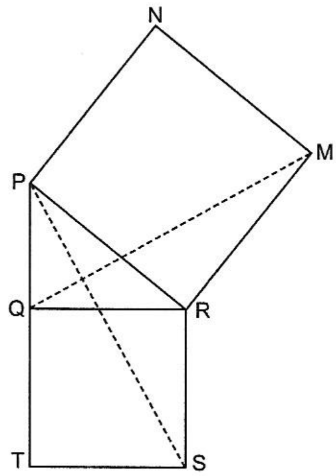
(*ii*) LM = LN, i.e. LMN is an isosceles triangle

[CBSE 2016]

13. In the given figure, RS = QT and QS = RT. Prove that PQ = PR.



14. In the given figure, ΔPQR is a right-angled triangle with ∠Q = 90°. If QRST is a square on side QR and PRMN is a square on hypotenuse of a right-angled triangle. Prove that PS = QM.



15. ABC is a triangle in which ∠ B = 2 ∠ C. D is a point on side BC such that AD bisects ∠ BAC and AB = CD. Prove that ∠ BAC = 72°.